MULTIMODE UTILITY THEORY:
A theory of rational subjectivity

Menlo Park, 2012

Louis Narens

Department of Cognitive Sciences
Department of Logic and Philosophy of Science
Institute for Mathematical Behavioral Sciences
University of California, Irvine

Rationality

There are many concepts of “rationality” for decision making under uncertainty.

Rationality

There are many concepts of "rationality" for decision making under uncertainty.

Some of the popular ones for economic modelings are:

- Dutch Book Argument (De Finetti, 1972)
- SEU (Savage, 1954)
- Conditional expected utility (Luce & Krantz, 1971)

Rationality

There are many concepts of “rationality” for decision making under uncertainty.

Some of the popular ones for economic modelings are:

- Dutch Book Argument (De Finetti, 1972)
- SEU (Savage, 1954)
- Conditional expected utility (Luce & Krantz, 1971)

They all imply that uncertainty has the structure of a boolean algebra and uncertainty is measured by a finitely additive probability function .

Objections to SEU as a Model of Rationality

Objections to SEU as a Model of Rationality

The main idea to the objections: SEU does not appropriately account for “risk”.

Objections to SEU as a Model of Rationality

The main idea to the objections: SEU does not appropriately account for "risk".

- Allais (1953)
- Ellsberg (1961)

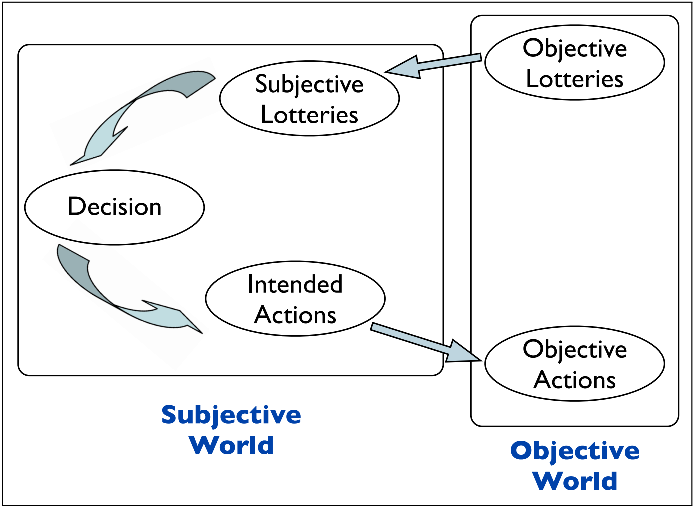
Objections to SEU as a Model of Rationality

The main idea to the objections: SEU does not appropriately account for "risk".

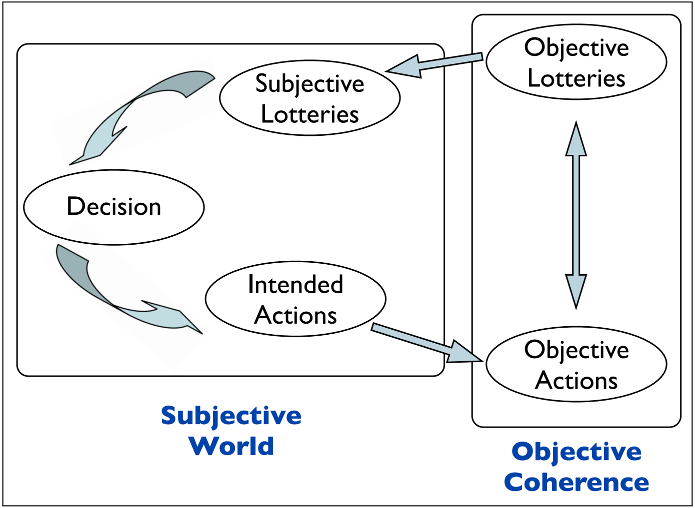
- Allais (1953)
- Ellsberg (1961)

A number of modifications of SEU for capturing various concepts of risk has been suggested in the economic literature. Some of these give up uncertainty being measured by a finitely additive probability function. They all, however, retain the boolean algebra structure of events.

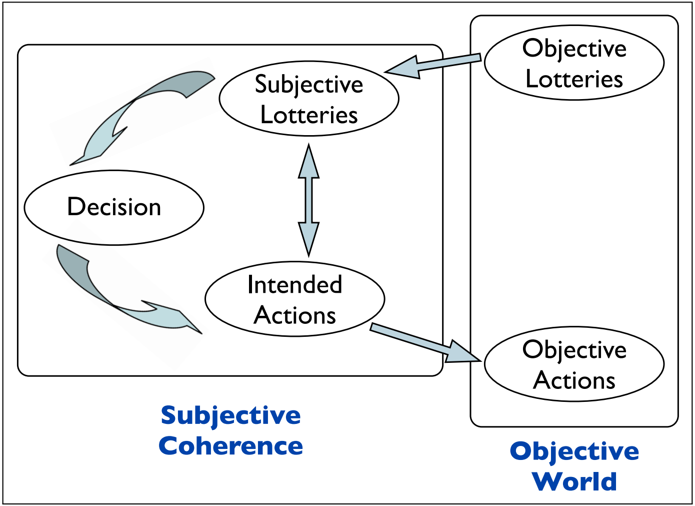
Decision Making with Subjective Stages



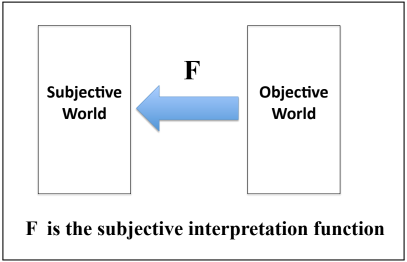
Objective Rationality



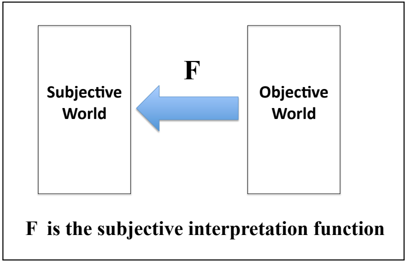
Subjective Rationality



Subjective Interpretation Function

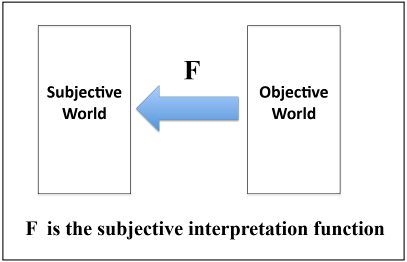


Subjective Interpretation Function



In general, subjective interpretations distort features of the objective world. For example, gambles may be distorted.

Subjective Interpretation Function



In general, subjective interpretations distort features of the objective world. For example, gambles may be distorted.

For the purposes of this lecture it will be assumed that *the subjective interpretations of gambles are themselves gambles.*

Kissing Your Favorite Movie Star Experiment

Rottenstreich & Hsee (2001) Experiment.

Kissing Your Favorite Movie Star Experiment

Rottenstreich & Hsee (2001) Experiment.

Certainty condition participants were asked to imagine that they could receive either "the opportunity to meet and kiss your favorite movie star" or $50 in cash.

Kissing Your Favorite Movie Star Experiment

Rottenstreich & Hsee (2001) Experiment.

Certainty condition participants were asked to imagine that they could receive either "the opportunity to meet and kiss your favorite movie star" or $50 in cash.

Low probability condition participants were asked to imagine that they could take part in either a lottery offering a 1% chance of winning "the opportunity to meet and kiss your favorite movie star" or a lottery offering a 1% chance of winning $50 in cash.

Kissing Your Favorite Movie Star Experiment

Rottenstreich & Hsee (2001) Experiment.

Certainty condition participants were asked to imagine that they could receive either "the opportunity to meet and kiss your favorite movie star" or $50 in cash.

Low probability condition participants were asked to imagine that they could take part in either a lottery offering a 1% chance of winning "the opportunity to meet and kiss your favorite movie star" or a lottery offering a 1% chance of winning $50 in cash.

Certainty condition: 70% of participants preferred the cash over the kiss.
Low-probability condition: 65% of participants preferred the kiss lottery over the cash lottery.

Rationality and Human Normative Behavior

Basic idea of this lecture: (1) Humans are emotional creatures and emotions should be taken into account in “rational decisions.”

Rationality and Human Normative Behavior

Basic idea of this lecture: (1) Humans are emotional creatures and emotions should be taken into account in "rational decisions."

Instead of arguing about term "rational", I will reformulate (1) as (1'): *Human normative behavior should be formulated to include human emotions.*

Normativeness and Rationality

Normativeness and Rationality

Figure: Normative

Normativeness and Rationality

Figure: Normative

Figure: Rational

Normative Behavior

Key idea of this talk: Normativeness for human decisions making under uncertainty should be formulated in terms of subjective coherence.

Boolean Event Spaces and Logic

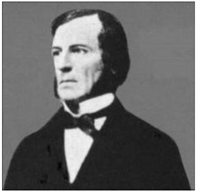


Figure: Boole

Boolean event spaces were originally developed for describing the algebra inherent in classical logic, which is the logic appropriate for Platonism.

Quantum Event Spaces and Logic

Figure: von Neumann

Von Neumann in the 1930's realized (i.e., proved to himself) that the phenomena in quantum mechanics could not be described using boolean logic and an boolean event space. He provided a probabilistic foundation for quantum mechanics using a different kind of event space.

Intuitionistic Mathematics


Figure: Brouwer and Heyting

Brouwer invented a radically new approach to mathematics based on the idea that mathematical concepts were mentally based. Roughly similar to the medieval conceptualist views about abstract concepts, except more detailed and clearly reasoned out.

Intuitionistic Mathematics

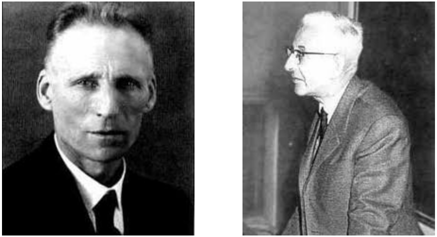


Figure: Brouwer and Heyting

Brouwer invented a radically new approach to mathematics based on the idea that mathematical concepts were mentally based. Roughly similar to the medieval conceptualist views about abstract concepts, except more detailed and clearly reasoned out.

Heyting formulated a logic that could account for the deductions that Brouwer made in his new radical mathematics. His axiomatization became known as *intuitionistic logic*.

Topological Semantics for Intuitionistic Logic

Figure: Raisowa and Sikorski

Rasiowsa and Sikorski developed an algebraic-topological approach to understanding classical and intuitionistic logic and its metatheory.

Topological Semantics for Intuitionistic Logic

Figure: Raisowa and Sikorski

Rasiowsa and Sikorski developed an algebraic-topological approach to understanding classical and intuitionistic logic and its metatheory.

The concepts and properties of event spaces used in my research are based on ideas and methods of Rasiowa and Sikorski.

Anticipation Event Spaces

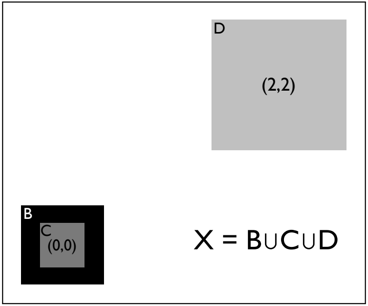
$\langle \mathcal{X}, \cup, \cap, \dot{-}, X, \varnothing \rangle$ is said to be an **anticipation event space** if and only if

- X and $\varnothing$ are in $\mathcal{X}$ and $X \neq \varnothing$
- all elements of $\mathcal{X}$ are subsets of X,
- for all A and B in $\mathcal{X}$, $A \cup B$ and $A \cap B$ are in $\mathcal{X}$,
- $\dot{-}$ is an operation on $\mathcal{X}$, called **pseudo complementation,** such that

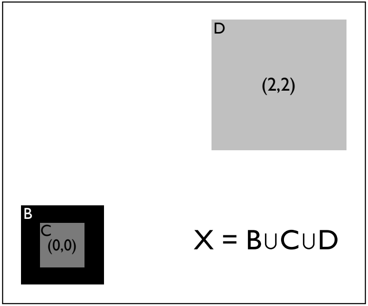
$$\dot{-}A \;=\; \text{the largest element } B \text{ of } \mathcal{X} \text{ such that } B \subseteq X - A\,,$$

i.e., if C is in $\mathcal{X}$ and $C \subseteq X - A$ then $C \subseteq B$.

Example of an Anticipation Algebra that is not Boolean



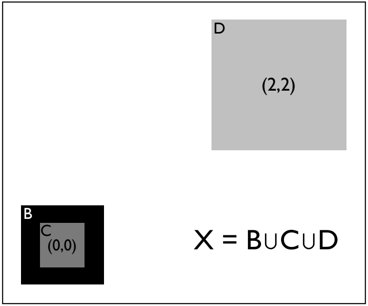
Example of an Anticipation Algebra that is not Boolean



The algebra consists of the events

$$\emptyset,\ B,\ C,\ D,\ C \cup D,\ X\ (= B \cup D = C \cup B \cup C)\,.$$

Example of an Anticipation Algebra that is not Boolean

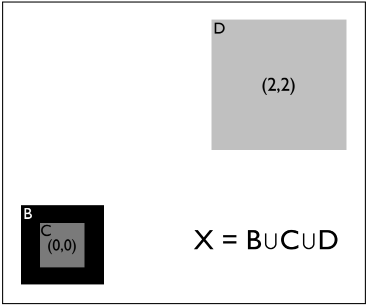


The algebra consists of the events

$$\varnothing,\ B,\ C,\ D,\ C \cup D,\ X\ (= B \cup D = C \cup B \cup C)\,.$$

The algebra is not boolean, because $B - C$ is not in the algebra.

Example of an Anticipation Algebra that is not Boolean



The algebra consists of the events

$$\varnothing,\ B,\ C,\ D,\ C \cup D,\ X\ (= B \cup D = C \cup B \cup C)\,.$$

The algebra is not boolean, because $B - C$ is not in the algebra.

Note that $\dot{-}C = \dot{-}B = D$ and that $\dot{-}D = B$. Thus, $C \neq B$, but $\dot{-}\dot{-}C = \dot{-}\dot{-}B$.

Probability and Topological Interpretation

If $\mathcal{T}$ is a topology with universe T, then

$$\langle \mathcal{T}, \cup, \cap, \dot{-}, T, \varnothing \rangle$$

is an anticipation event space.

Probability and Topological Interpretation

If $\mathcal{T}$ is a topology with universe T, then

$$\langle \mathcal{T}, \cup, \cap, \dot{-}, T, \varnothing \rangle$$

is an anticipation event space.

Suppose $\langle \mathcal{X}, \cup, \cap, \dot{-}, X, \varnothing \rangle$ is an anticipation event space. Then $\mathbb{P}$ is said to be a *probability function* on $\mathcal{X}$ if and only if $\mathbb{P}$ is a function from $\mathcal{X}$ into $[0, 1]$ such that

- $\mathbb{P}(X) = 1 \quad \& \quad \mathbb{P}(\varnothing) = 0$,
- for all A and B in $\mathcal{X}$,

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)\,.$$

Descriptions

A typical description of the lottery

$$L = (a_1, A_1; \ldots; a_i, A_i; \ldots; a_m, A_m)$$

has the form,

$$\mathbf{L} = (\mathbf{a}_1, \mathbf{A}_1; \ldots; \mathbf{a}_i, \mathbf{A}_i; \ldots; \mathbf{a}_m, \mathbf{A}_m) .$$

L is called the *objective interpretation* of L and is denoted by $\mathcal{I}_O(\mathbf{L})$.

Similarly, the *objective interpretations* of $\mathbf{A}_i$ and $\mathbf{a}_i$ are, respectively, A_i and a_i, and are denoted by, respectively, $\mathcal{I}_O(\mathbf{A}_i)$ and $\mathcal{I}_O(\mathbf{a}_i)$.

Interpretations

$\mathcal{M}$ denotes the set of states of the decision maker. o, the special state of objectivity, may or may not be in $\mathcal{M}$.

SEU has a decision maker with just one state—the objective state.

Interpretations

$\mathcal{M}$ denotes the set of states of the decision maker. o, the special state of objectivity, may or may not be in $\mathcal{M}$.

SEU has a decision maker with just one state—the objective state.

In MUT (*Multimode Utility Theory)*, the decision maker can be in many states and her interpretation of a description can vary with with state.

Interpretations

$\mathcal{M}$ denotes the set of states of the decision maker. o, the special state of objectivity, may or may not be in $\mathcal{M}$.

SEU has a decision maker with just one state—the objective state.

In MUT (*Multimode Utility Theory)*, the decision maker can be in many states and her interpretation of a description can vary with with state.

$\mathcal{I}_i(\mathbf{A})$ denotes the decision maker's interpretation of $\mathbf{A}$ when she is in state M. Similarly for $\mathcal{I}_M(\mathbf{a})$ and $\mathcal{I}_M(\mathbf{L})$.

DSEU (*Descriptive Subjective Utility*) Models

DSEU models are a special subcase of MUT models.

DSEU (*Descriptive Subjective Utility*) Models

DSEU models are a special subcase of MUT models.

• The event space for its probability function, P, is an anticipation event space. *(A MUT restriction)*

DSEU (*Descriptive Subjective Utility*) Models

DSEU models are a special subcase of MUT models.

• The event space for its probability function, P, is an anticipation event space. *(A MUT restriction)*

• The formula

$$u(\mathbf{L}) = \sum_{i=1}^{m} \frac{\mathrm{P}[\mathcal{I}_{M_i}(\mathbf{A}_i)]u(\mathbf{a}_i)}{\mathrm{P}[\mathcal{I}_{M_1}(\mathbf{A}_1)] + \cdots + \mathrm{P}[\mathcal{I}_{M_m}(\mathbf{A}_m)]},$$

for computing utility. *(A DSEU restriction)*

DSEU (*Descriptive Subjective Utility*) Models

DSEU models are a special subcase of MUT models.

• The event space for its probability function, P, is an anticipation event space. *(A MUT restriction)*

• The formula

$$u(\mathbf{L}) = \sum_{i=1}^{m} \frac{\mathrm{P}[\mathcal{I}_{M_i}(\mathbf{A}_i)]u(\mathbf{a}_i)}{\mathrm{P}[\mathcal{I}_{M_1}(\mathbf{A}_1)] + \cdots + \mathrm{P}[\mathcal{I}_{M_m}(\mathbf{A}_m)]},$$

for computing utility. *(A DSEU restriction)*

• The formula

$$M_i = F(\mathbf{a}_i, \mathbf{A}_i)$$

for specifying the decision maker's state. *(A DSEU restriction)*

DSEU (*Descriptive Subjective Utility*) Models

DSEU (*Descriptive Subjective Utility*) Models

- If $\mathcal{I}_M(\mathbf{A})$ and $\mathcal{I}_N(\mathbf{A})$ are defined, then

$$\dot{-}\dot{-}\mathcal{I}_M(\mathbf{A}) = \dot{-}\dot{-}\mathcal{I}_N(\mathbf{A})\,.$$

(A MUT restriction)

DSEU (*Descriptive Subjective Utility*) Models

- If $\mathcal{I}_M(\mathbf{A})$ and $\mathcal{I}_N(\mathbf{A})$ are defined, then

$$\dot{-}\dot{-}\mathcal{I}_M(\mathbf{A}) = \dot{-}\dot{-}\mathcal{I}_N(\mathbf{A}) \, .$$

(A MUT restriction)

A consequence of the above is that if $\mathcal{I}_M(\mathbf{A})$ and $\mathcal{I}_N(\mathbf{B})$ are defined and $A \cap B = \varnothing$, then $\mathcal{I}_M(\mathbf{A}) \cap \mathcal{I}_N(\mathbf{B}) = \varnothing$.

DSEU (*Descriptive Subjective Utility*) Models

- If $\mathcal{I}_M(\mathbf{A})$ and $\mathcal{I}_N(\mathbf{A})$ are defined, then

$$\dot{-}\dot{-}\mathcal{I}_M(\mathbf{A}) = \dot{-}\dot{-}\mathcal{I}_N(\mathbf{A})\,.$$

(A MUT restriction)

A consequence of the above is that if $\mathcal{I}_M(\mathbf{A})$ and $\mathcal{I}_N(\mathbf{B})$ are defined and $A \cap B = \varnothing$, then $\mathcal{I}_M(\mathbf{A}) \cap \mathcal{I}_N(\mathbf{B}) = \varnothing$.

Thus the interpretation of a lottery remains a lottery with the outcomes in the same order.

DSEU Modeling

DSEU modeling is intended for formulating and testing a theory, not for direct modeling of data. In particular, the function F, where $M_i = F(\mathbf{a}_i, \mathbf{A}_i)$, must be stated before the modeling of data.

DSEU Modeling

DSEU modeling is intended for formulating and testing a theory, not for direct modeling of data. In particular, the function F, where $M_i = F(\mathbf{a}_i, \mathbf{A}_i)$, must be stated before the modeling of data.

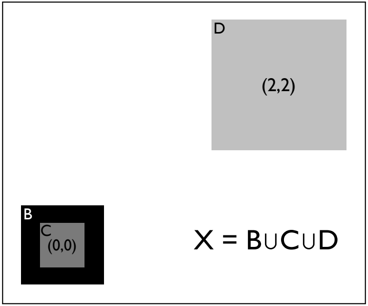
When this, and other DSEU restrictions are met, then DSEU models generally have fewer or the same number parameters than other models for the same kind of cognitive decision phenomenon.

Example: Kissing Your Favorite Movie Star Experiment

Example: Kissing Your Favorite Movie Star Experiment

Low probability condition participants were asked to imagine that they could take part in either a lottery offering a 1% chance of winning "the opportunity to meet and kiss your favorite movie star" or a lottery offering a 1% chance of winning $50 in cash.

T1 $=$ **(Kiss,Winning Ticket; \$0,Losing Ticket)**, **T2** $=$ **(\$50,Wining Ticket; \$0,Losing Ticket)**.



Kiss in **T1** puts the decision maker into mode M and $\mathcal{I}_M$(**Winning Ticket**) $= B$; **\$50** in **T2** put the decision maker into mode N and $\mathcal{I}_N$(**Winning Ticket**) $= C$; **\$0** in both **T1** and **T2** puts the decision maker in mode N and $\mathcal{I}_N$(**Losing Ticket**) $= D$.

Example: Fear

Chichilnisky (2009) presents a generalization of classical subjective expected utility theory that emphasizes the special role that catastrophes play in human decision processes.

Her intent is "to define rational behavior more broadly, and more in tune with the way humans behave."

Fear

In this example $\mathcal{M}$ has two modes, a rational mode, R, and a fear mode F, which are related as follows: : For all lottery event descriptions $\mathbf{A}$,

$$\mathcal{I}_F(\mathbf{A}) = [\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] = [\dot{-}\dot{-}\mathcal{I}_{\mathcal{O}}(\mathbf{A})] \supset \mathcal{I}_R(\mathbf{A}) = \mathcal{I}_{\mathcal{O}}(\mathbf{A}) = A\,.$$

As a consequence, $\mathrm{P}[\mathcal{I}_F(\mathbf{A})] > \mathrm{P}[\mathcal{I}_R(\mathbf{A})]$.

Fear

In this example $\mathcal{M}$ has two modes, a rational mode, R, and a fear mode F, which are related as follows: : For all lottery event descriptions $\mathbf{A}$,

$$\mathcal{I}_F(\mathbf{A}) = [\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] = [\dot{-}\dot{-}\mathcal{I}_{\mathcal{O}}(\mathbf{A})] \supset \mathcal{I}_R(\mathbf{A}) = \mathcal{I}_{\mathcal{O}}(\mathbf{A}) = A\,.$$

As a consequence, $\mathrm{P}[\mathcal{I}_F(\mathbf{A})] > \mathrm{P}[\mathcal{I}_R(\mathbf{A})]$.
The outcome **a** in the term description (**a**,**A**) puts the decision maker into one of the states R or F.

Fear

$$\mathcal{I}_F(\mathbf{A}) = [\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] = [\dot{-}\dot{-}\mathcal{I}_{\mathcal{O}}(\mathbf{A})] \supset \mathcal{I}_R(\mathbf{A}) = \mathcal{I}_{\mathcal{O}}(\mathbf{A})\,.$$

This raises the question about how states in $[\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ are to be interpreted. Three approaches are considered:

Fear

$$\mathcal{I}_F(\mathbf{A}) = [\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] = [\dot{-}\dot{-}\mathcal{I}_{\mathcal{O}}(\mathbf{A})] \supset \mathcal{I}_R(\mathbf{A}) = \mathcal{I}_{\mathcal{O}}(\mathbf{A})\,.$$

This raises the question about how states in $[\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ are to be interpreted. Three approaches are considered:

(1) Fear makes apparent to the decision maker what are additional states that could yield the catastrophic outcome.

Fear

$$\mathcal{I}_F(\mathbf{A}) = [\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] = [\dot{-}\dot{-}\mathcal{I}_{\mathcal{O}}(\mathbf{A})] \supset \mathcal{I}_R(\mathbf{A}) = \mathcal{I}_{\mathcal{O}}(\mathbf{A})\,.$$

This raises the question about how states in $[\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ are to be interpreted. Three approaches are considered:

(1) Fear makes apparent to the decision maker what are additional states that could yield the catastrophic outcome.

(2)The adding of states in $[\dot{-}\dot{-}\mathcal{I}_F(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ to those of $\mathcal{I}_R(\mathbf{A})$ is a mathematical way of increasing the decision makers perceived probability of $\mathbf{A}$. The states in $[\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ are *fictitious* (called *virtual* in some literatures).

Fear

$$\mathcal{I}_F(\mathbf{A}) = [\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] = [\dot{-}\dot{-}\mathcal{I}_{\mathcal{O}}(\mathbf{A})] \supset \mathcal{I}_R(\mathbf{A}) = \mathcal{I}_{\mathcal{O}}(\mathbf{A}).$$

This raises the question about how states in $[\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ are to be interpreted. Three approaches are considered:

(1) Fear makes apparent to the decision maker what are additional states that could yield the catastrophic outcome.

(2)The adding of states in $[\dot{-}\dot{-}\mathcal{I}_F(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ to those of $\mathcal{I}_R(\mathbf{A})$ is a mathematical way of increasing the decision makers perceived probability of $\mathbf{A}$. The states in $[\dot{-}\dot{-}\mathcal{I}_R(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ are *fictitious* (called *virtual* in some literatures).

(3) The adding of states in $[\dot{-}\dot{-}\mathcal{I}_F(\mathbf{A})] - \mathcal{I}_R(\mathbf{A})$ to those of $\mathcal{I}_R(\mathbf{A})$ is a result of the decision maker's beliefs: Fear produces illusions involving the existence of such states, and she takes these illusions into account in the computation of $u(\mathbf{L})$.

Subjective Rationality of DSEU

SEU is use in many places in science as the gold standard for rationality.

Subjective Rationality of DSEU

SEU is use in many places in science as the gold standard for rationality.

A theorem shows the following: Each DSEU model is isomorphically imbeddable in a SEU model.

Subjective Rationality of DSEU

SEU is use in many places in science as the gold standard for rationality.

A theorem shows the following: Each DSEU model is isomorphically imbeddable in a SEU model.

(1) I take this theorem as showing that DSEU **within the subjective world** has the **same formal properties** that are used to characterize rationality in the objective world.

(2) This alone does not imply that DSEU will pass pragmatic tests carried out in the **objective world** of rationality designed for SEU.

Conclusion

Humans are emotional and subjective.

Conclusion

Humans are emotional and subjective.

In traditional decision theories, emotion and subjectivity are treated like a crazed person in need of a straight jacket.

Conclusion

Humans are emotional and subjective.

In traditional decision theories, emotion and subjectivity are treated like a crazed person in need of a straight jacket.

In SEU, the straight jacket takes the form of a rigid structure that only allows subjectivity for the importance of outcomes and the size of probabilities. Generally the sizes of probabilities are highly constrained by symmetry and other structural considerations. Other decision theories, to keep a rational appearance, cut SEU into separate pieces and re-tailor them into a slightly less stifling garment.

Conclusion

Humans are emotional and subjective.

In traditional decision theories, emotion and subjectivity are treated like a crazed person in need of a straight jacket.

In SEU, the straight jacket takes the form of a rigid structure that only allows subjectivity for the importance of outcomes and the size of probabilities. Generally the sizes of probabilities are highly constrained by symmetry and other structural considerations. Other decision theories, to keep a rational appearance, cut SEU into separate pieces and re-tailor them into a slightly less stifling garment.

In contrast, the decision theory discussed in this lecture, DSEU, sees instead a colorful individual in need of a new kind of clarity to reveal his or her rational intent.