

# Conflict, the Harsanyi-Zeuthen bargaining solution, and departures from expected utility

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May 28, 2012

## Abstract

Under RDEU axiomatics, we show that optimistic preferences, as defined by a concave probability distortion function, lead to a greater likelihood of conflictual behavior within the Harsanyi-Zeuthen bargaining framework, while pessimistic preferences, as defined by a convex probability distortion function, increase the likelihood of a negotiated settlement. The Harsanyi-Zeuthen framework would therefore appear to offer a simple and appealing manner in which to introduce non-expected utility considerations into the analysis of conflict.

## 1 Introduction

Conflicts and warfare with their high costs and dubious rewards are particularly difficult to rationalize even though they continue to occur. Fear appears to be a powerful motivator for such extreme behavioral responses in general and for conflict and violence in particular. These appear irrational at the outset and cannot usually be explained through standard models of decision making such as expected utility as been shown convincingly by Chichilnisky (2010).<sup>1</sup> In this paper we want to reestablish what constitutes in our view the critical explanatory role of emotions in the understanding of conflict initiation and conflict persistence. We also want to show how attitudes towards risk and uncertainty can lead to and stabilize cooperation. Our approach is at variance with more classical analyses despite the fact that very eminent authors have recognized the importance of emotions and attitudes toward risk in the examination of rational decision-making under incomplete information.<sup>2</sup> Fearon (1995) in his paper on “rationalist explanations for war” undertakes an investigation of the same question, which is centered on decisions to initiate war or not. Since Fearon addresses directly the question of how to explain fighting as opposed to bargaining and presents a slightly more recent version of the puzzle and of the traditional ways to resolve it, we will take his presentation of the problem as a point of departure to develop our own quite different perspective.

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<sup>1</sup>She suggests an alternative axiomatization of utility theory in order to account for attitudes involving fear of catastrophes.

<sup>2</sup>See, for instance Harsanyi and Selten (1988), p. 10.

What we call a traditional way to consider this problem is a conception that takes for granted: (i) that decision-makers are essentially risk neutral, occasionally weakly risk averse and (ii) that probabilities about such crucial outcomes such as winning wars or the costs associated with them (another random variable) are essentially given by nature and are not based on subjective estimates by the parties involved. The same can be said about the selection of types (usually hard or soft) that protagonists to a conflict are supposed to represent: They are assumed to be given once and for all by a random draw out of a predetermined probability distribution. The resulting theory of conflict and war is relatively simple: war may occur because two parties do not have complete information about each other's means or each other's preferences. Moreover, within the framework of incomplete information, fighting involves taking short-term losses but may help creating a reputation that will keep opponents away in the future. While such reputation building is certainly plausible, it is sometimes hard to understand when very weak powers nevertheless confront very strong ones (e.g. the Israel Palestinian conflict and the two successive Intifadas). Moreover, to work, it requires the introduction of the notion of commitment, which is supposed to overcome the reluctance decision-makers might have for costly actions. Why certain categories of decision makers choose to commit and others do not is again left to a random selection of types left to nature. Beyond this however, the existing theory is unable to explain three major aspects of conflict interactions: (i) why long term conflict occurs, (ii) escalation processes and (iii) why cooperation fails to lead to optimal results.

In what follows we want to challenge this by now dominant view. While acknowledging its explanatory powers we want to draw attention to other, in our view more straightforward conceptualizations. We want to present resolutions of the puzzle that are based on attitudes decision makers exhibit about risk taking and result from the poor understanding they have about each other in this regard. Fearon's approach based on the work of earlier game theorist has been called the bargaining model of conflict and this is in our view a correct presentation of the basic issue: to explain a choice for a chancy action such as war, its superiority over a bargained outcome has to be established. However, the links between defecting from a bargain to choose conflict in Fearon's work and classical bargaining theory from Nash to Harsanyi to Rubinstein are not investigated, even though such relations are in our view very important. Our analysis intends thus also to reestablish the deep connection between theories about choices of either conflict initiation or the pursuit of negotiations and bargaining approaches. As we will show, such a perspective allows us not only to put conflict and war initiation into a rational framework but also to explain how opponents can remain trapped in a conflict outcome.

To situate our conception with respect to bargaining theory a short description of it is appropriate. Traditional bargaining theory has been presented within two apparently different, but as we will see ultimately common frameworks.<sup>3</sup> The first and older conception is due to John Nash (1950, 1953). Nash showed that a unique solution to a two person

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<sup>3</sup>Because the Nash solution sometimes involves the use of cooperative strategies, it is often considered to be only a part of cooperative game theory. However, the Nash theory is not confined to cooperative games. Harsanyi, for instance, has a lengthy discussion about its pertinence for non-cooperative games. See Harsanyi (1977):273-290. This point is also emphasized by Hargreaves Heap and Varoufakis (1995):113.

bargaining problem obtains under conditions of (1) joint efficiency; (2) symmetry of gains to the two actors if the game situation they were involved in was symmetric; (3) linear invariance of the solution; and (4) independence of the solution from irrelevant alternatives.<sup>4</sup> The unique solution to the joint bargaining problem is the result of maximizing the Nash product.

In 1956, Harsanyi showed that Nash's theory is mathematically equivalent to an earlier theory of bargaining due to Frederik Zeuthen (1930).<sup>5</sup> Harsanyi showed how the Zeuthen theory expresses the bargaining process as a sequence of moves that eventually converge to the Nash bargaining solution. This Harsanyi-Zeuthen mechanism is based upon the concept of a *critical risk ratio*. The critical risk ratio measures the probability of defecting or choosing the conflict outcome.<sup>6</sup> It is given by:

$$r_i = \frac{U_i(x_{ij}) - U_i(x_{ji})}{U_i(x_{ij}) - U_i(c)},$$

where  $x_{ij}$  represents what agent  $i$  expects from agent  $j$  in the bargaining process,  $x_{ji}$  is what he gets as an offer from  $j$  and  $c$  represents the value of no agreement or conflict between the two agents. For agent  $i$ , it is immediate that  $r_{ij} = 0$  if the offer from agent  $j$  corresponds exactly to what he wants. On the other hand,  $r_{ij} = 1$  if the offer from the other side does not differ from the value of the conflict situation. Thus  $r_{ij}$  varies between 0 and 1. A symmetric consideration holds for agent  $j$ . Zeuthen further postulated that:

- Rule 1: within a bargaining process, the player with the lower critical risk ratio makes a concession to the player with the higher critical risk ratio such that the inequality is reversed.
- Rule 2: when the critical risk ratios of both players are equal, both make a concession.

We will refer to these as the Harsanyi-Zeuthen rules. Harsanyi shows that if both agents behave in this way, the bargaining process will converge to the Nash solution. While Rule 1 seems fairly plausible, Rule 2 seems less so and appears to be rather *ad hoc*. As such, Harsanyi (1956) showed that it could be derived from first principles. More recently, the bargaining approach advanced by Rubinstein (1982, 1985) has been considered more convincing than the Harsanyi-Zeuthen-Nash approach.

On the surface, these two approaches look very different. While the Harsanyi-Zeuthen-Nash theory can be interpreted as a sequence of bargaining moves, the particular sequential nature of the bargaining process is not taken into account. The Rubinstein conception is explicitly built on a process of alternating offers and counter-offers at different moments in time according to the following script: Agent  $C$  makes an offer at time 1, to agent  $R$ , for a division of a certain good. The amount of the good is assumed to be fixed so that if the offer

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<sup>4</sup>A thorough presentation of the Nash postulates is presented in Harsanyi (1977):144-146 and also in Binmore (1998):94-98.

<sup>5</sup>In particular, see his Chapter IV.

<sup>6</sup>While Harsanyi (1977) applies critical risk in a neutral context, Ellsberg (1961a) applies it in a non-cooperative game situation.

made by  $C$  is  $x$ , then  $(1 - x)$  would be left to  $R$ . The bargaining process is characterized by time discounting: as time goes on, the value of the good shrinks at a different rate for each agent. This discounting and the sequential nature of the bargaining process favors the agent who makes the first offer since rejecting an offer is costly for the other agent. Rubinstein (1982) shows that if the first agent anticipates in his first offer the discounting of the other agent with respect to successive offers and counter-offers, then his initial offer will be accepted.<sup>7</sup>

## 2 The Harsanyi-Zeuthen critical risk ratio and conflict

### 2.1 Expected utility axiomatics

#### 2.1.1 The critical risk ratio and the risk premium

Our point of departure is the observation that the condition which yields conflict in the Fearon (1995) setup trivially corresponds to a Harsanyi-Zeuthen critical risk ratio condition. To see this, consider a simple two state of nature model. With probability  $p$ , party 1 loses, yielding utility  $U(L)$ , whereas she win with probability  $1 - p$ , yielding utility  $U(W)$ . Party 1 will thus engage in conflict when:

$$pU(L) + (1 - p)U(W) > U(S), \quad (1)$$

where  $S$  is the negotiated ("sure thing") outcome. It is immediate that this expression can be rewritten as the Harsanyi-Zeuthen critical risk ratio condition:

$$\frac{U(W) - U(S)}{U(W) - U(L)} = r > p. \quad (2)$$

The expression given in (1) also allows one to establish a simple link between the incentives for conflict and risk attitudes. Define the *actuarially fair* risk premium in the usual manner (Pratt 1964) as  $\pi$  such that:

$$pU(L) + (1 - p)U(W) - U(pL + (1 - p)W - \pi) = 0. \quad (3)$$

Then conflict obtains whenever  $pU(L) + (1 - p)U(W) = U(pL + (1 - p)W - \pi) > U(S)$ . Since  $U(\cdot)$  is strictly increasing, the inequality corresponds to  $pL + (1 - p)W - \pi > S$ , which implies that the Harsanyi-Zeuthen critical risk ratio condition can be written as:  $\frac{W - S - \pi}{W - L} > p$ . For a strictly concave  $U(\cdot)$ , a trivial application of Jensen's inequality (Pratt 1964) yields  $\pi > 0$ , which implies that:

$$r = \frac{W - S - \pi}{W - L} > p.$$

The opposite obtains under a convex utility function.

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<sup>7</sup>The Rubinstein perspective is well described in Osborne and Rubinstein (1990).

### 2.1.2 Implications of unobservable preferences for conflict

If these different ratios are now replaced within the framework of the Harsanyi-Zeuthen theory, the bargaining process between two agents evolves through concessions that are driven by the relations between their respective critical risk probabilities  $r_i$  and  $r_j$ . If one now assumes incomplete information about utilities, then  $\pi_i$  is essentially unobservable by agent  $j$  and vice versa, one can nevertheless postulate that an agent can reasonably anticipate the subjective risk ratio  $r_i = \frac{W_i - S_{ji} - \pi_i}{W_i - L_i}$  on the basis of the more “objective” ratio

$$\hat{r}_i = \frac{W_i - S_{ji}}{W_i - L_i}.$$

This estimate is based upon an objective evaluation of gains and losses and the offer that the agent makes himself to the other.

What does this mean for the bargaining process? It implies that under risk aversion (concave  $u(\cdot)$ ), with  $\hat{r}_i > r_i$ , there exists, according to the Harsanyi-Zeuthen rules, an incentive to offer a greater concession to the other side than if the subjective  $r_i$  were directly observable. It follows then that under risk aversion ( $\pi_i, \pi_j > 0$ ), mutual concessions will obtain and thus that the cooperative outcome will be reached within the Harsanyi-Zeuthen process. In some sense, both agents are risk anticipating. Conversely, under risk preference ( $\pi_i, \pi_j < 0$ ),  $\hat{r}_i < r_i$ , the bargainers underestimate the critical risk ratio and thus do not concede, even when they make more and more stringent requests to the other side. They end therefore, through the same Harsanyi-Zeuthen process in reverse, at the conflict outcome. In this case agents are risk debasing. We summarize this intuition in the following Proposition:

**Proposition 1** *Assume the Harsanyi-Zeuthen bargaining framework, and assume that agent  $i$  ( $j$ ) estimates agent  $j$ 's ( $i$ 's) critical risk ratio by  $\hat{r}_j$  ( $\hat{r}_i$ ). Then, when  $\pi_i, \pi_j > 0$ , conciliation obtains. Conversely, when  $\pi_i, \pi_j < 0$ , a negotiated settlement is impossible and conflict breaks out.*

**Proof.** To see why, all we have to show is that the use of the opponent's objective risk ratio instead of the (“true”) subjective risk ratio is consistent with the Harsanyi-Zeuthen principles. First, note that  $j$  estimates agent  $i$ 's risk ratio through  $\hat{r}_i > r_i$ . Thus, when  $r_i > r_j$ , it will necessarily also be the case that  $\hat{r}_i > r_j$  and  $j$  will make a concession to  $i$  that reverses the inequality in the Harsanyi-Zeuthen framework. But when  $\hat{r}_i < r_j$  it is also the case that  $r_i < \hat{r}_i < r_j < \hat{r}_j$  so when  $i$  estimates  $j$ 's risk ratio using  $\hat{r}_j$ ,  $i$  will make a concession to  $j$  such that  $\hat{r}_j < r_i$ . Iterating this process leads to the standard Harsanyi-Zeuthen result. Now suppose that  $r_i = r_j$ . In this case  $i$  estimates  $j$ 's risk ratio through  $\hat{r}_j > r_j$  which implies that  $r_i = r_j < \hat{r}_j$  and  $i$  makes a concession to  $j$ . A similar reasoning holds for  $j$ , and we are back in the standard Harsanyi-Zeuthen framework in which both players make concessions and the Nash bargaining solution obtains. The opposite obtains under risk preference. To see why, there are three initial configurations to consider. Configuration 1: suppose that  $r_i = r_j$ . If  $i$  estimates his opponent's risk ratio through  $\hat{r}_j$ , it will be the case that  $\hat{r}_j < r_i = r_j$  and  $i$  will *not* make a concession to  $j$ . A similar line of reasoning applies to  $j$  and conflict will obtain. Configuration 2: now assume that  $\hat{r}_j < r_i < r_j$ : the

same reasoning applies and conflict obtains. Configuration 3: there remains the case in which  $r_i < \hat{r}_j < r_j$ . In this case,  $i$  will offer a concession to  $j$  such that the first inequality is reversed, yielding  $\hat{r}_j < r_i$ ; but if this is the case, we are back in Configuration 2 ( $\hat{r}_j < r_i < r_j$ ) and conflict obtains. ■

Proposition 1 lead to two conclusions. On the one hand, if both parties are risk-averse, conflict and war initiation is exceptional since powerful incentives exist to concede to the other side. However, if both parties are risk-loving, the equilibrium involves conflict. Thus this analysis leads to the existence of a conflict or war trap, an outcome that is then very difficult to overcome except under complete surrender of one side or by the exhaustion of all resources.

Proposition 1 shows that as long as we can sign the risk premium, we can –using the Harsanyi-Zeuthen process– predict the outcome of the negotiation between the parties. It follows that it suffices to sign the risk premium, irrespective of the axiomatics that generate the parties preferences, for us to be able to predict the outcome of the bargaining process. This is in contrast to recent approaches to the bargaining problem using alternative axiomatics, in which the Nash bargaining solution has remained elusive. For example, Köbberling and Peters (2003) are only able to integrate Rank-Dependent Expected Utility (RDEU) preferences in a straightforward manner within the confines of the Kalai-Smorodinsky bargaining solution, with the Nash bargaining approach yielding unappealing (non-regular) behavior. Similarly, while Koskiewic (1999) is able to integrate RDEU preferences within an axiomatic Nash bargaining framework, his results only hold for a convex probability transformation function –which, as we shall see below, corresponds only to pessimistic attitudes.<sup>8</sup>

## 2.2 RDEU axiomatics

The Rank-Dependent Expected Utility (RDEU) model was initially developed by Quiggin (1982) in order to address a number of important weaknesses that had become apparent in the EU approach.<sup>9</sup> Under RDEU, the linearity in probabilities of the EU model is replaced by a probability weighting, perception, or distortion function (see Chateauneuf, Cohen, and Meilijson (2005)) which assigns weights to the probabilities of the different states of nature, where the weights are themselves functions of the rank of the given state of nature, in terms of the level of satisfaction that the individual derives. An important contribution of the RDEU approach has been to separate risk attitudes, on the one hand, from the marginal utility of income, on the other: in the usual EU approach, the marginal utility of income is forced into playing both roles.

Under Rank-Dependent Expected Utility axiomatics,  $U : \mathbb{R} \rightarrow \mathbb{R}$ , defined up to a monotone increasing transformation, plays the role of a utility function under certainty, and  $\varphi : [0, 1] \rightarrow [0, 1]$ , which satisfies the restrictions  $\varphi(0) = 0$  and  $\varphi(1) = 1$ , is a unique

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<sup>8</sup>See also Rubinstein, Safra, and Thomson (1992), Safra and Zilcha (1993) and Burgos, Grant, and Kajii (2002). Shalev (2002) considers the case of loss-aversion, which (as would be the case with Cumulative Prospect Theory) poses additional problems because of the need to endogenize the status quo or "reference" point.

<sup>9</sup>The most celebrated of which is probably the Ellsberg (1961b) paradox.

function that plays the role of a probability transformation function;  $U$  and  $\varphi$  are both continuous and increasing. One of the great attractions of RDEU axiomatics, apart from resolving a number of problems inherent in EU, is that it allows one to rigorously define the concepts of *optimism* and *pessimism*. Optimism corresponds to  $\varphi$  being *concave*, while pessimism corresponds to  $\varphi$  being *convex*.<sup>10</sup>

In an RDEU framework, the equivalent to equation (1) is given by:

$$U(L) + \varphi(1-p)[U(W) - U(L)] > U(S). \quad (4)$$

Intuitively speaking, RDEU axiomatics correspond to a situation in which agents are certain of receiving at least the worst outcome  $U(L)$ , and perceive a distorted probability  $\varphi(1-p)$  of achieving the higher outcome  $U(W)$ , leading to a gain over the worst outcome equal to  $U(W) - U(L)$ . When  $\varphi(\cdot)$  is given by the identity function, one gets the special case of EU.<sup>11</sup>

Straightforward manipulations of (4) then yield the modified Harsanyi condition as:

$$\frac{U(W) - U(S)}{U(W) - U(L)} > 1 - \varphi(1-p), \quad (5)$$

which of course boils down to the condition given in (2) when  $\varphi(\cdot)$  corresponds to the identity function.

We first recall the following simple Proposition:

**Proposition 2 (Courtault and Gayant, 1998)** *A second-order approximation to the risk premium under RDEU axiomatics is given by  $\pi_\varphi = [(1-p) - \varphi(1-p)](W - L) + \frac{\sigma_\varphi^2}{2}A(E)$ , where  $E$  is the expectation of the gain,  $\sigma_\varphi^2$  is the distorted variance, and  $A(E)$  is the Arrow-Pratt coefficient of absolute risk aversion.*

**Proof.** We begin by recalling the standard Pratt second-order approximation to the risk premium. Let  $E = pL + (1-p)W$  represent the expectation of the gain from conflict. Consider the following three Taylor expansions:  $U(L) \approx U(E) + (L - E)U'(E) + \frac{1}{2}(L - E)^2U''(E)$ ,  $U(W) \approx U(E) + (W - E)U'(E) + \frac{1}{2}(W - E)^2U''(E)$ ,  $U(E - \pi) \approx U(E) - \pi U'(E)$ . Substitution of these three expressions into (3) then yields  $\pi = \frac{\sigma^2}{2}A(E)$ , where  $\sigma^2 = p(L - E)^2 + (1-p)(W - E)^2$  is the variance of the gain from conflict, and  $A(E) = -\frac{U''(E)}{U'(E)}$  is the Arrow-Pratt coefficient of absolute risk aversion evaluated at the

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<sup>10</sup>For experimental evidence on the actual shape taken by the probability weighting function  $\varphi$ , see Camerer and Ho (1994), Tversky and Fox (1995) and Prelec (1998).

<sup>11</sup>In mathematical terms, the RDEU preference functional is simply a Choquet (1953) integral, rather than the usual Lebesgue-Stieltjes integral of expected utility theory. In essence, as shown in Theorem 7 of Wakker (1990),  $\varphi(1-p)$  is the functional form taken by Choquet's more general *capacities*. While one can take  $\varphi(\cdot)$  to be a component of the agents' risk preferences in a RDEU context, the results that follow can also be reinterpreted in terms of Choquet-Schmeidler expected utility (CEU, Gilboa (1987), Schmeidler (1989)), where a convex (concave)  $\varphi(\cdot)$  corresponds to ambiguity aversion (preference).

expectation of the gain from conflict. Strict concavity of the utility function then implies that  $\pi > 0$ . Strict convexity leads to  $\pi < 0$ . Now consider the RDEU case. Let the actuarially fair risk premium  $\pi_\varphi$  under RDEU axiomatics be defined in the usual manner by:  $U(L) + \varphi(1-p)[U(W) - U(L)] - U(E - \pi_\varphi) = 0$ . Then, using the same Taylor expansions as above yields:  $\pi_\varphi = [(1-p) - \varphi(1-p)](W-L) + \frac{\sigma_\varphi^2}{2}A(E)$ , where  $\sigma_\varphi^2 = [1 - \varphi(1-p)](L-E)^2 + \varphi(1-p)(W-E)^2 > 0$  is the "distorted" variance of the gain from conflict, and  $A(E) = -\frac{U''(E)}{U'(E)}$  is the Arrow-Pratt coefficient of absolute risk aversion evaluated at the expectation of the gain from conflict. ■

Proposition 2 is simply a specialization to the two states of nature case of the more general result provided by Courtault and Gayant (1998). As in section 2.1, conflict breaks out when:

$$r_\varphi = \frac{W-S-\pi_\varphi}{W-L} > p,$$

where  $r_\varphi$  is the Harsanyi-Zeuthen critical risk ratio under RDEU axiomatics (which depends upon the probability distortion function  $\varphi(\cdot)$ ).

### 2.2.1 The Yaari functional

It is immediate that when the utility function is linear, we are in a situation that corresponds to Yaari's (1987) dual theory functional.<sup>12</sup> In this case, since  $U''(\cdot) = 0$ , the expression given for the risk premium in Proposition 2 simplifies to:

$$\pi_\varphi = [(1-p) - \varphi(1-p)](W-L). \quad (6)$$

This immediately leads to the following Proposition:

**Proposition 3** *Let agent preferences be described by the Yaari (1987) functional. Then: (i) when agents are pessimistic  $\pi_\varphi > 0$ ,  $r_\varphi = \frac{W-T-\pi_\varphi}{W-L} < \frac{W-T}{W-L} = \hat{r}$  and conciliation is more likely under the Harsanyi-Zeuthen bargaining framework; (ii) when agents are optimistic  $\pi_\varphi < 0$ ,  $r_\varphi = \frac{W-T-\pi_\varphi}{W-L} > \frac{W-T}{W-L} = \hat{r}$  and conflict is more likely under the Harsanyi-Zeuthen bargaining framework.*

**Proof.** Under RDEU axiomatics, pessimism corresponds to a convex probability distortion function  $\varphi(\cdot)$ ; it follows that, since  $1-p \in (0,1)$ ,  $(1-p) > \varphi(1-p)$ , which implies that  $\pi_\varphi > 0$ . The converse obtains under optimism, since  $\varphi(\cdot)$  is then concave. ■

Proposition 3 yields the intuitively appealing result that optimism is associated with more aggressive behavior (and a greater likelihood of conflict), while pessimism leads to more conciliatory behavior, and that this is true even in a context in which marginal utility is constant (as in the original Fearn model).

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<sup>12</sup>In the special case of the Yaari functional, in which the utility function is linear, Roell (1987), Demers and Demers (1990), and Guriev (2001) show that the Choquet expectation corresponds to a simple Lebesgue-Stieltjes integral. The differentiability of the Yaari functional has recently been studied by Carlier and Dana (2003).

### 2.2.2 Decreasing or increasing marginal utility

In the general case in which marginal utility is not constant, we can establish the following Proposition.

**Proposition 4** *Let agent preferences be described by the Quiggin (1982) RDEU functional, and let marginal utility be decreasing. Then: (i) if preferences are pessimistic, it will always be the case that  $\pi_\varphi > 0$  and conciliation is more likely under the Harsanyi-Zeuthen bargaining framework; (ii) if preferences are optimistic and  $\pi_\varphi > 0$ , conciliation is more likely under the Harsanyi-Zeuthen bargaining framework; (iii) if preferences are optimistic and  $\pi_\varphi < 0$ , conflict is more likely under the Harsanyi-Zeuthen bargaining framework.*

**Proof.** (i) With a strictly concave utility function and pessimistic preferences, it will always be true that  $\pi_\varphi > 0$ , which immediately yields the result; (ii) this corresponds to a situation in which  $0 < [(1 - p) - \varphi(1 - p)](W - L)$  and  $[(1 - p) - \varphi(1 - p)](W - L) + \frac{\sigma_\varphi^2}{2}A(E) > 0$ , implying that  $\pi_\varphi > 0$ ; (iii) if  $[(1 - p) - \varphi(1 - p)](W - L) < 0$  and  $[(1 - p) - \varphi(1 - p)](W - L) + \frac{\sigma_\varphi^2}{2}A(E) < 0$ , it will be the case that  $\pi_\varphi < 0$ , which immediately yields the result. ■

Proposition 4 (i) is the equivalent of Proposition 3 (i): under pessimistic preferences risk-aversion and pessimism work in the same direction, with both inducing more conciliatory behavior. Proposition 4 (ii) and (iii) show that, with optimistic preferences and decreasing marginal utility, the outcome depends upon the relative magnitudes of the effects due to the optimistic probability distortion function on the one hand, and decreasing marginal utility on the other. When the latter dominates, we obtain the same result as in Proposition 4 (i); when the former dominate, we obtain the same result as in Proposition 3 (ii).

One can, of course, extend Proposition 4 to the case of risk-loving behavior. In this case, it is immediate that optimistic preferences will always be associated with a greater likelihood of conflict, while the outcome under pessimistic preferences will depend upon the degree of convexity of the utility function, as was the case in parts (ii) and (iii) of Proposition 4 for the case of a concave utility function.

## 3 Concluding remarks

The results presented in this paper mirror those of Volij (2002), in the context of Rubinstein's (1982) game with time preference, in which he showed that dynamically consistent non-expected utility maximizers (who satisfy the compound independence axiom) yielded the Rubinstein result, and was indistinguishable from the behavior of expected utility maximizers. The difference between our approach and that of Volij, apart from placing ourselves within the Harsanyi-Zeuthen framework, is that we allow for a modicum of asymmetric information in that each player cannot observe the other's true critical risk ratio. The basic message of our paper, however, is clear: when preferences can be characterized as being optimistic, conflict is more likely. When preferences are pessimistic, conflict is less likely and a negotiated outcome will obtain.

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