AFOSR Workshop on

Catastrophic Risks

BLISS, CATASTROPHE, AND RATIONAL POLICY

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I. FROM NICHOLAS BERNOULLI TO KARL MENGER AND BEYOND

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I. FROM NICHOLAS BERNOULLI TO KARL MENGER AND BEYOND

A. **Pascal et. al.** Evaluation of games by expected value. Pascal's wager.

B. **Nicholas Bernoulli's letter**: "St. Petersburg" game. $P(n) = 2^{-n}$, $X_n = 2^n$. E(X) infinite. Published in Rémond's book (1713).

C. **Gabriel Cramer** (1728): Letter to Nicholas Bernoulli. Expected utility. $U = x^{1/2}$; also bounded.

D**. Daniel Bernoulli** (1738; St. Petersburg Academy) General view of expected utility as a guide to action. Especial emphasis on $U = \ln X$.

E. **1739-1933**: Expected utility mentioned with respect (e.g., Marshall) but no application I know of and no critical analysis.

F. **F.P. Ramsey (1931):** Utility in intertemporal optimization. Wanted to avoid "bliss," i.e., the existence of infinite value for a feasible consumption stream. $U' = c^{-\eta}$, $\eta > 1$.

G. **Karl Menger (1934):** Suppose U unbounded. Then St. Petersburg-like paradox. Choose X_n so that $U(X_n) = 2^n$, $P(n) = 2^{-n}$. Then E(U) infinite. Menger took a negative additude to EU

H. **von Neumann and Morgenstern (1944, 1947):** Axiomatization of EU. Includes an Archimedean axiom (continuity)

II. THE ROLE OF CONTINUITY

A. **Compound Gambles**: Indifferent to simple gamble which gives the same probabilities of outcomes

B. **The Meaning of Continuity**: A preference relation is said to be continuous if the upper and lower contour sets are closed, i.e., if x_{ni} is preferred to y, all n, and $\{x_n\}$ approaches x_0, then x_0 is at least as good as y, and the same if the preference is reversed. Let G be a gamble with an infinite expected utility, let x_0 and x_1 be any two definite outcomes with x, preferred to x_0, and let G'(p) be the compound gamble yielding G with probability p and x_0 with probability 1-p. Then, for any $p > 0$ (no matter how small), G'(p) is preferred to x_1, no matter how poor an outcome x_0 is or how good x_1 is. But G'(0) means getting x_0 with probability 1, which is, by construction, inferior to x_1.

C. **Bliss and Catastrophe**: I have followed the literature in discussing the problems with a utility function unbounded above. But the problems are identical with functions unbounded below and negative infinities, which we may call, "catastrophes."

D. **Everyday Life and the Avoidance of Catastrophe:** Is death a catastrophe? Value of statistical life is finite. Other values: the nation, social approval, money.

III. GENERALIZED HYPERBOLIC UTILITY

A. **Conditions for Bounded Utility:** Let,

$$\eta(c) = - c\, U''(c)/U'(c).$$

For U to be bounded below it is necessary that $\eta(c) < 1$ for c sufficiently small; to be bounded above, it is necessary that $\eta(c) > 1$ for c sufficiently large. This suggests the stronger condition of increasing relative risk aversion (IRRA), that,

$\eta(c)$ increasing, $\eta(0) < 1$, $\eta(\text{infinity}) > 1$.

B. **A Simple Bounded Utility Function**

Let,

$$\eta(c) = \alpha\,[1 - (1-\beta)\,\gamma\ (c + \gamma)^{-1}],$$

where $\beta, \gamma > 0$, $\alpha\beta < 1 < \alpha$. Then, $\eta(0) = \alpha\beta < 1$, $\eta(\text{infinity}) = \alpha > 1$. The marginal utility function implied by this is,

$$U'(c) \text{ proportional to } [(c + \gamma)^{1-\beta}\ c^{\beta}]^{-\alpha}.$$

Computing optimal intertemporal paths with and without uncertainty is fairly easy.

IV. SOME POLICY IMPLICATIONS

A. **Weitzman's "Dismal Theorem":** Weitzman argued that, if the temperature sensitivity (in a climate change model) is uncertain and if this uncertainty is represented by a fat-tailed distribution (asymptotic to a power function at infinity), then any policy which leads to an increase in CO_2 concentration has an infinite expected marginal utility. More important, it would lead to a negative infinity of expected utility. He assumed $\eta(c)$ contant and > 1.

B. **Implications**: Sacrifice everything to prevent any increase in CO_2 concentration.