1. Introduction

Global environmental phenomena like climate change, major extinction events or flu-type pandemics can have catastrophic consequences. By properly assessing the outcomes involved – especially those concerning human life – economic theory of choice under uncertainty is expected to help people take the best decision. However, the widely used expected utility theory values life in terms of the low probability of death someone would be willing to accept in order to receive extra payment. Common sense and experimental evidence refute this way of valuing life, and here we provide experimental evidence of people’s unwillingness to accept a low probability of death, contrary to expected utility predictions. This work uses new axioms of choice, especially an axiom that allows extreme responses to extreme events, and the choice criterion that they imply. The implied decision criteria are a combination of expected utility with extreme responses, and seem more consistent with observations.

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valuing human life, since EU theory does not “fit” with the stated behavior of most of the subjects in the experiment.

This paper provides a theoretical framework by considering death as a ‘catastrophe’, namely a rare event with major consequences. Using the new axioms of choice introduced in Chichilnisky (2000, 2002), we derive a choice criterion that is more consistent with the experimental evidence on how people value catastrophic events such as death. We show that EU theory underestimates rare events and that this originates from the classic axiom of continuity (Monotone Continuity, defined in Arrow, 1971) which implies that rational behavior involves insensitivity to rare events with major consequences like death. We replace the axiom of continuity by an alternative axiom of sensitivity to rare events, formalizing a theory of choice under uncertainty where rare but catastrophic events (such as death) are given a treatment in symmetry with the treatment of frequent events. As a consequence, a probability can be considered low enough to make the lottery involving death acceptable; it all depends on what the other outcomes are.

This implies a different way of valuing life, one that seems more in tune with experimental evidence. First, this new way of valuing life is in keeping with evidence provided by the experiment reported below, given that age and family situation appear to affect the way subjects change their decisions about whether or not to take action impacting the value of their lives. More generally, it may explain why in some experiments people appear to give unrealistically high numerical values to life that are not consistent with the empirical evidence about how they choose occupations, for instance. Second, this new way of valuing life is in keeping with evidence provided by experimental psychologists, who observe that the brain reacts differently when making a decision involving rare situations inspiring extreme fear (LeDoux, 1996). Overall, the proposed framework suggests an alternative way to define rational behavior when catastrophic risks are involved.

The remainder of the paper proceeds as follows. Section 2 presents the experimental evidence. Section 3 recalls recent contributions in the literature on modeling risk and catastrophic events, shows how EU theory fails to appropriately value life and proposes a solution. The final Section discusses the results and draws conclusions.

2. Experimental Evidence

We present the results of an experiment (referred to below as the pill experiment) which twice asked a sample of subjects a question implying a trade-off between the risk of dying and a fixed amount of money, at an interval of 11 years.

2.1. The 1998 Initial Pill Experiment

In February 1998, the members of a Research Center in Quantitative Economics were asked by internal e-mail (in French): “Imagine that you are offered the opportunity to play a game in which you must choose and swallow one pill out of 1 billion (10^9) identical pills. Only one contains a lethal poison that is sure to kill you, all the other pills being ineffective. If you survive (i.e. you swallow one of the 999,999,999 ineffective pills), you receive a tax-free amount of €152,450. Are you willing to choose one pill and to swallow it?”.

The value subjects attribute to their own life can be assessed using the classic utility theory of choice under uncertainty. Indeed, state-dependent models, simple single period models, life-cycle models when the change in mortality lasts over an infinitesimally short time (Johansson, 2003) as well as wage-risk trade-off models for marginal changes in risk (see Rosen, 1988; Viscusi, 1993) rely on the EU theory and express the VPF as a marginal rate of substitution between wealth and risk of death. What happens if this approach is crudely applied to the results of the above experiment?

Before answering, it should be pointed out that studies aiming at valuing life never ask the kind of direct question we use. They generally use either data from market choices that involve an implicit trade-off between risk and money (labor or housing markets, portation, self-protection or averting behaviors), or stated preferences elicited in more subtle ways and using unidentified victims. Moreover, stated preferences suffer from limitations, both generally and in this case: the actual behavior is not observed; due to incorrect sensitivity to probabilities, smaller changes in risk tend to induce higher VPF estimates (Beattie et al., 1998); a significant gap exists between willingness to pay and willingness to accept...

Finally, the lack of monetary incentives in this experiment may puzzle the reader and is briefly justified below. A number of authors (e.g. Smith, 1976; Harrison, 1994, or Smith and Walker, 1993) emphasize the importance of paying subjects in real cash and providing appropriate monetary incentives in experiments, based on the principle that monetary incentives are needed to motivate people sufficiently when answering hypothetical questions and that this leads to better performance. On the contrary, other authors, including social (and economic) psychologists (Loewenstein, 1999; Slovic, 1969; Tversky and Kahneman, 1992), consider that subjects should be intrinsically motivated enough to answer truthfully in the experiment and that social or affective incentives may be even better motivators than monetary incentives.

This is a controversial issue among researchers, regularly raised by new experiments or meta-analyses. A case in point is Camerer and Hogarth (1995), who analyzed 74 experiments either known to them (1953–1998) or published in famous US journals from 1990 to 98. These studies all varied incentives substantially. The authors found no effect on mean performance in most of the studies (though variance is usually reduced by higher payment) and noted that “no replicated study has made rationality violations disappear purely by raising incentives”. They conclude that apart from cases in which subjects are required to make a major cognitive effort and/or face an incentive to lie, monetary incentives are not mandatory.

Neither of these conditions applies to our experiment, which moreover has several characteristics suggesting that subjects were intrinsically motivated to answer truthfully: they were volunteer colleagues, with a potential reciprocity concern vis-à-vis the experimenter; they were told they would be provided with a summary of the experimental results; the topic can be considered entertaining and of intellectual interest; and the experiment was not time-consuming at all (5 min). We are therefore confident that participants answered seriously even without monetary incentives, which would have been difficult to implement in this case.

All that being said, subjects face a choice between compensation (€152,450) for accepting a change in risk of death (increase of 10^-15) and a status quo alternative. Subjects who answer ‘Yes’ clearly consider that €152,450 is enough to compensate for the increase in death risk, whereas those who answer ‘No’ do not. Due to the referendum-type elicitation question, the minimum amount at which subjects would accept the increase in risk is unknown. Among the 64 responses collected, 33 subjects answered ‘No’ and 31 answered ‘Yes’ (see the second column of Table 1 for details by answer type).

Do some subjects’ characteristics explain such behavior? We look for dependences between the answer given and individual characteristics with contingency chi-square tests (see the second column of Table 2). No evidence of dependence is found: the p-values are far from the usual significant levels in use. These results are confirmed by performing an analysis of variance for main effects and crossed

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1 Note that the original wording mentioned FRF 1,000,000. In 1998, the exchange rate was 1 USD per 5.9 FRF.

2 However, in our experiment, the victim, although identified, is only exposed to an (infinitesimal) risk change, not to certain death.
given) graduate courses in Statistics), one plausible explanation is that more than
one cannot correctly understand low probabilities (all belong to a Quanti-
tative variables in binomial discrete choice models (Probit and Contingency chi-square tests (p-values).

effects (interactions): no characteristic appears significant because subjects give different answers in 1998 and 2009.

Table 1
Composition of the samples (in %).

<table>
<thead>
<tr>
<th>Sample</th>
<th>1998 survey (n=64)</th>
<th>2009 survey (n=120)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Gender</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>Position</td>
<td>Ph.D. student</td>
<td>Admin. Staff</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net individual income</td>
<td>&lt;€1500/month</td>
<td>€1500–€2499/month</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parenthood</td>
<td>No children</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>At least one child</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Change in Parenthood between 1998 and 2009.</td>
<td></td>
</tr>
</tbody>
</table>

3 Change in Parenthood between 1998 and 2009.

2.2. The 2009 Follow-up Pill Experiment

In January 2009, the same question was again put by e-mail to the initial 1998 sample as well as to new members of the same Research Center.\(^3\) Examining the motivations for their answer is crucial, and to this end, they were questioned on the influence various factors had on their answer. Subjects then gave a mark on a scale of 0 to 5 (where ‘0’ equates to ‘no influence at all’ and ‘5’ equates to ‘very strong influence’) to the following changes in factors: marital/familial status (Family), financial status (Financial), health status (Health), age (Age), life expectancy (LifeExp), perception of the probability (PercProb), opinion w.r.t. this type of issue (Opissue), relation to chance (Chance), relation to death (Death). An open question at the end allowed subjects to state other factors (Other) or give open comments.

Of the 64 initial members, 2 had unfortunately died, it was impossible to find a way of contacting 3 at the time of the study, and 2 did not answer e-mails. The 2009 sample is thus composed of 57 out of 64 (89%) initial members and 63 new members, i.e. a total of 120 subjects.

We then test whether 1998 subjects answer differently from 2009 subjects. The standard test consists in comparing the proportion of ‘Yes’ (or ‘No’) in the two samples. However, we should take into account that the samples overlap since 57 subjects belong to both samples. We then use a test of equal proportion that accounts for that (in particular for the fact that the variance of the two proportions is the same under the null hypothesis, see Bland and Butland). The proportion of ‘No’ for the 2009 sample (n=120) significantly exceeds that for the 1998 sample (n=64, two-sample proportion-comparison test with ‘Bland and Butland’ p-value = .0246). If we restrict to the overlapping subjects (n=57), we obtain the same result (one-sample proportion-comparison test with ‘standard’ p-value = .0297). Let us consider now the two sub-samples that answer the 2009 follow-up experiment: the 57 subjects that previously answered the 1998 experiment and the 63 new subjects. The proportions of ‘No’ for these two sub-samples do not significantly differ (two-sample proportion-comparison test with ‘standard’ p-value = .1776).

Finally, let us focus on the 57 subjects that answered both 1998 and 2009 surveys. Moving from the aggregate level to the individual level,
we observe that 15 subjects changed their mind between 1998 and 2009: 12 by switching from 'Yes' to 'No' and 3 from 'No' to 'Yes'. Five of them gave open comments to explain what motivates their change "I take much bigger risks in everyday life without such a high reward, so I have decided to change from 'No' to 'Yes'.", 'I now have two children and do not want to add any additional risk -- however tiny it may be -- that may have painful implications for their life" ('Yes' to 'No'), 'My position on the consequences my death would have for my relatives has changed" ('Yes' to 'No'), "I am now married with twins, fully happy and I want nothing more" ('Yes' to 'No') and "the 1998 ratio of gain variation over risk variation expressed in French Francs (i.e., \(10^6/10^{-6}\)) was more attractive than the current ratio expressed in euros (\(182,000/10^{-6}\)) w.r.t. the probability perception, even though the monetary gains are similar in terms of purchasing power" (Yes to No). Note that in 9 changes out of 15, the subject had had one (or several) child(ren) since 1998.

Among these 57 subjects, descriptive statistics by change in answer are shown in the fourth column of Table 1 (with 'Yes' for a change and 'No' for no change). Dependences between a change in answer and individual characteristics were also tested with contingency chi-square tests (see last column of Table 2) and do not want to add any additional risk

### Table 3

Descriptive statistics on mark by motivation (n = 120).

<table>
<thead>
<tr>
<th>Motivation</th>
<th>Mean</th>
<th>Std.-Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
<th># non null</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optisue</td>
<td>2.15</td>
<td>2.14</td>
<td>0</td>
<td>5</td>
<td>68</td>
</tr>
<tr>
<td>PercProba</td>
<td>2.01</td>
<td>2.21</td>
<td>0</td>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>Family</td>
<td>1.83</td>
<td>2.13</td>
<td>0</td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td>Death</td>
<td>1.75</td>
<td>2.07</td>
<td>0</td>
<td>5</td>
<td>58</td>
</tr>
<tr>
<td>Financial</td>
<td>1.63</td>
<td>1.85</td>
<td>0</td>
<td>5</td>
<td>64</td>
</tr>
<tr>
<td>Chance</td>
<td>1.21</td>
<td>1.94</td>
<td>0</td>
<td>5</td>
<td>43</td>
</tr>
<tr>
<td>Age</td>
<td>1.05</td>
<td>1.70</td>
<td>0</td>
<td>5</td>
<td>43</td>
</tr>
<tr>
<td>LifeExp</td>
<td>1.01</td>
<td>1.62</td>
<td>0</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>Health</td>
<td>.82</td>
<td>1.43</td>
<td>0</td>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>Other</td>
<td>4.62</td>
<td>0.74</td>
<td>3</td>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>

Note that Arrow (1971), p. 257, introduces the axiom of continuity (MC) and Hernstein and Milnor (1953) call it Axiom 2. However continuity depends on the notion of 'closeness' that is used. A monotone decreasing sequence of events \(E_i\), \(i = 1, 2, 3, ...\), is a sequence for which all \(i, E_i \subseteq E_{i+1}\). If there is no state in the world common to all members of the sequence, \(\cap_i E_i = \emptyset\) and \(E_i\) is called a vanishing sequence. For example, in the case of the real line, the sequence \((n, \infty)\), \(n = 1, 2, 3, ...\), is a vanishing sequence of sets.

In Arrow (1971), two lotteries\(^5\) are close to each other when they have different consequences in small events, which he defines as "An
event that is far out on a vanishing sequence is ‘small’ by any reasonable standards” and more formally, as follows:

Axiom of Monotone Continuity (MC) Given a and b, where a > b, a consequence c, and a vanishing sequence (E\textsuperscript{i}), suppose the sequences of actions (a\textsuperscript{i}), (b\textsuperscript{i}) satisfy the conditions that (a\textsuperscript{i}, s\textsuperscript{i}) satisfy the same consequences as (a, s) for all s in (E\textsuperscript{c}) and the consequence c for all s in E, while (b\textsuperscript{i}, s\textsuperscript{i}) yields the same consequences as (b, s) for all s in (E\textsuperscript{c}) and the consequence c for all s in E. Then for all i sufficiently large, a\textsuperscript{i} > b and a\textsuperscript{i} > b’ (Arrow, 1971, p. 48).

In Arrow’s framework, two lotteries that differ in sets of small enough Lebesgue measure are very close to each other.

On the basis of the standard axioms of choice (including MC), a crucial result established that individuals optimize the ranking of lotteries WE\textsubscript{U} according to an expected utility function. The expected utility of a lottery f is a ranking defined by WE\textsubscript{U}(f) = \int_{R} f(x)\mu(x) where \mu is a measure with an integrable density function \phi(.) that belongs to the class of all measurable and integrable functions on R so \mu(A) = \int_{A} \phi(x)dx, where dx is the standard Lebesgue measure on R. The ranking WE\textsubscript{U}(\cdot) is a continuous linear function that is defined by a countably additive measure \mu\textsuperscript{6}.

3.2. Recent Contributions to the Modeling of Catastrophic Risks

In recent work, a catastrophic risk is described as an event that has “a very low probability of materializing, but that if it does materialize will produce a harm so great and sudden as to seem discontinuous with the flow of events that preceded it” (Posner, 2004, p. 6). This interpretation of catastrophic risks is entirely consistent with ours. However, Posner (2004) does not model decisions with catastrophic risks — he refers to EU analysis and points out that this analysis is in-adequate to explain the decisions that people make when confronted with catastrophic risks.

The modeling of catastrophic risks in Weitzman (2009) is based on EU and assumes that there are “heavy tails” (defined as distributions that have an infinite moment generating function). He seeks to explain behavioral discrepancies by attributing them to these unexplained “heavy tails”. It should be noted that, “heavy tails” being inconsistent with the main axioms of EU, this leads to the non-existence of a robust solution and unacceptable conclusions, like using all the current resources to mitigate future catastrophes. As noted by Buchholz and Schymura (2010), a model assuming both EU theory and utility function unbounded below leads to catastrophic events playing a dominant role in the decision-making process. In contrast, the choice of utility functions that are bounded below leads to implausibly low degrees of relative risk aversion and catastrophic events playing no role in the decision-making process.

To avoid infinite values, Weitzman (2011) suggests thinning or truncating the probability distribution, or putting a cap on utility. Other authors try to reconcile EU and “heavy tails” by using specific utility functions other than power Constant Relative Risk Aversion utility. For instance, Ikeeji et al. (2010) propose the two-parameter Burr function or exponential utility function and Millner (2011) proposes the (bounded) Harmonic Absolute Risk Aversion function to model individual preferences. However, this approach yields results driven by subjective choices like functional forms or parameter values, which is not fully satisfactory, as Weitzman (2011) admits.

Chichilinsky’s (1996, 2000, 2002, 2009) approach, presented hereafter, differs from the above in proposing a systematic axiomatic foundation for modeling catastrophic risks or for making decisions when risks are catastrophic. She shows why EU theory fails to explain half the answers in the experiment and identifies the MC axiom as the source of the problem, proposing an alternative set of axioms that appears to fit the experimental evidence.

3.3. The Failure of EU and a Solution

The failure of EU appears to be due to the MC axiom which implicitly postulates that rational behavior should be ‘insensitive’ to rare events with major consequences. More specifically, the culprit is the underlying definition of proximity that is used in the MC axiom, where two events are close to each other when they differ on a set of small measure no matter how great the difference in their outcomes. Chichilinsky (2000, 2002, 2009) used a L\textsuperscript{∞} sup norm that is based on extreme events to define closeness: two lotteries f and g are close when they are uniformly close almost everywhere (a.e.), i.e. when supR|f(t) - g(t)| < \epsilon for a suitable small \epsilon > 0.7 As a consequence, some catastrophic events are small under Arrow’s definition but not necessarily under Chichilinsky’s.

The core here is that her definition of closeness is more sensitive to rare events than Arrow’s. It implies that a probability can be considered as low enough to make the lottery involving death acceptable, depending on what the other outcomes are. This higher sensitivity constitutes the second of her three axioms, which must be satisfied by a ranking W to evaluate lotteries:

Axiom 1. The ranking W: L\textsubscript{∞} \rightarrow R is linear and continuous on lotteries.

The ranking W is called continuous and linear when it defines a linear function on the utility of lotteries that is continuous with respect to the norm in L\textsubscript{∞}.

Axiom 2. The ranking W: L\textsubscript{∞} \rightarrow R is sensitive to rare events.

A ranking function W: L\textsubscript{∞} \rightarrow R is called insensitive to rare events when it neglects low probability events; formally if given two lotteries (f, g) there exists \epsilon = \epsilon(f, g) > 0 such that W(f) > W(g) if and only if W(f') > W(g') for all f', g' satisfying f' = f and g' = g a.e. on A \subset R when \mu(A') < \epsilon. We say that W is sensitive to rare events, when W is not insensitive to low probability events.

Axiom 3. The ranking W: L\textsubscript{∞} \rightarrow R is sensitive to frequent events.

Similarly, W: L\textsubscript{∞} \rightarrow R is said to be insensitive to frequent events when for every two lotteries f, g there exists \epsilon = \epsilon(f, g) > 0 such that W(f) > W(g) if and only if W(f') > W(g') for all f', g' such that f' = f and g' = g a.e. on A \subset R when \mu(A') > 1 - \epsilon. We say that W is sensitive to frequent events when W is not insensitive to frequent events.

Our notion of ‘nearby’ is stricter and requires that the lotteries be close almost everywhere, which implies sensitivity to rare events. Chichilinsky (2009) proved that EU theory fails to explain the behavior of individuals facing catastrophic events since:

Theorem 1. A ranking of lotteries W(\cdot): L\textsubscript{∞} \rightarrow R satisfies the Monotone Continuity Axiom if and only if it is insensitive to rare events (see proof in Chichilinsky, 2009).

A formal statement of the theorem is hence MC ⇔ ¬Axiom 2. The simple example below shows why the axiom MC leads to insensitivity to rare events.

Example. Assume that the Axiom MC is satisfied. By definition, this implies for every two lotteries f, g, every outcome c and every vanishing sequence of events (E\textsuperscript{i}) there exists N such that arbitrarily altering the outcomes of lotteries f and g on event E, where i > N, does not alter the ranking, namely f > g where f' and g' are the altered versions of lotteries f and g respectively.8 In particular since, for any given f and g, Axiom MC applies to every vanishing sequence of events (E\textsuperscript{i}), we can choose a sequence of events consisting of open intervals I = (I\textsuperscript{i})\textsubscript{i=1} such that f' = \{x \in R: x > i\} and another f =

\footnotesize{\textsuperscript{6} A ‘countably additive’ measure is defined in Appendix.}

\footnotesize{\textsuperscript{7} A similar topology was used in Debreu’s (1953) formulation of Adam Smith’s Invisible Hand theorem.}

\footnotesize{\textsuperscript{8} For simplicity, we consider alterations in those lotteries that involve the ‘worst’ outcome c = inf\{a(f(x), g(x)): x\} which exists because f and g are bounded a.e. on R by assumption.}
Theorem 2. A ranking of lotteries $W$ that satisfies Axiom 2 (i.e., that is sensitive to rare events) determines a value of life that changes depending on outcomes other than the amount of money.

Proof. Theorem 1 showed that sensitivity to rare events is the negation of the MC Axiom. This implies that for two given lotteries $f > g$, every outcome $c$ and every vanishing sequence of events $\{E_i\}$ there exists $N$ such that arbitrarily altering the outcomes of lotteries $f$ and $g$ on event $E_i$, where $i > N$, does not alter the ranking, namely $f^i > g^i$, where $f^i$ and $g^i$ are the altered versions of lotteries $f$ and $g$ respectively. For some other lotteries $f > g$, every outcome $c$ and some vanishing sequence of events $\{E_i\}$ there exists $N$ such that arbitrarily altering the outcomes of lotteries $f$ and $g$ on event $E_i$, where $i > N$, does alter the ranking, namely $g^i > f^i$, where $f^i$ and $g^i$ are the altered versions of lotteries $f$ and $g$ respectively. Recall that - as in Theorem 1 above - we have considered alterations in the lotteries that involve the ‘worst’ outcome $c = \inf_{x \in I} [f(x), g(x)]$. The worst outcome can be identified with “death”, and is common to the two sets of lotteries under comparison. The alterations in both cases are the same, representing small enough probabilities of death. This implies that, for a small enough probability of death (determined by $N$) in the first two lotteries, the subject will accept the risk of death when offered the small payment, while in the second lottery the same probability of death (represented by $N$) is not small enough. In other words, depending on other outcomes of the lotteries, the subject will accept a small probability of death, or will not accept that probability. Therefore, the value of life depends on other outcomes of the lottery, as we wished to establish.

Do some decision criteria satisfy all three Axioms in the presence of rare events? Yes, if we modify EU by adding another component called ‘purely finitely additive’ elements of $L_\infty$, that embodies the notion of sensitivity for rare events. The only acceptable rankings $W$ under the three axioms above are a convex combination of $L_1$ function plus a purely finitely additive measure putting all weight on extreme or rare events, as stated in the Theorem below:

Theorem 3. A ranking of lotteries $W: L_\infty \rightarrow R$ satisfies all three Axioms 1, 2 and 3, and if and only if there exist two continuous linear functions on $L_\infty$, $\phi_1$ and $\phi_2$ and a real number $\lambda$, $0 < \lambda < 1$, such that:

$$W_{\text{TRA}}(f) = \int_{L_\infty} \phi_1(x) dx + (1 - \lambda) \int_{L_\infty} \phi_2(x) dx$$

where $\int_{L_\infty} \phi_1(x) dx = 1$, while $\phi_2$ is a purely finitely additive measure (see proof in Chichilnisky, 1996, 2000, 2002).

The intuition behind this Theorem is that the first term of the utility in (1) is akin to EU, and therefore introduces a measure of sensitivity to normal or relatively frequent events. The density $\phi_1(x)$ defines a countably additive measure that is absolutely continuous with respect to the Lebesgue measure.

The second term of the utility in (1) is inconsistent with the MC axiom, and satisfies a different type of continuity, under a topology called “The Topology of Fear” (Chichilnisky, 2009). This second term is very sensitive to rare events and balances out the first term in the characterization, which is only sensitive to normal or frequent events. The operator $\int_{L_\infty} \phi_2(x) dx$ represents the action of a measure $\phi_2 \in L_\infty$ that differs from the Lebesgue measure in placing full weight on rare events. Remember that $\phi_2$ cannot be represented by an $L_1$ function.

The two terms together therefore satisfy both ‘sensitivity to rare events’, and ‘sensitivity to frequent events’, as is required by the new axiomatic treatment of decision making under uncertainty.
with catastrophic risks used here. The implied decision criteria that emerge from the new axioms are a combination of EU with extreme responses (to extreme events like death), and seem more in line with experimental evidence.

Indeed, it seems that purely finitely additive measures could play an important role in explaining how our brains respond to extreme risks. When the number of choices is finite there is a simpler way to explain the criterion of choice: *it is similar to a convex combination of EU and a maximin*. EU is optimized while at the same time avoiding those choices that involve catastrophic outcomes, such as death. This rule is inconsistent with EU and will rank a choice that involves death much lower than EU would. Therefore any observer that anticipates EU optimization will be disappointed, and will believe that there is irrationality. But this is not true, as the rule becomes rational once we take into account rational responses to extreme events. It is consistent with what people do on an everyday basis, with what is observed in the experiments presented here and also with what Arrow’s famous comment implies.

4. Discussion and Concluding Remarks

How the Axiom MC creates insensitivity to rare events can be illustrated by the following situation used by Arrow (1966) to show how people value their lives, along the same lines as the discussion in the Introduction. If a is an action that involves receiving one cent, b is another that involves receiving zero cents, and c is a third action involving receiving one cent and facing a low probability of death, Arrow’s Monotone Continuity requires that the third action involving death and one cent should be preferred to the second involving zero cents when the probability of death is low enough. Even Arrow says of his requirement ‘this may sound outrageous at first blush...’ (Arrow, 1966, p. 258, l. 28–29). Outrageous or not, we saw in Theorem 1 that MC leads to the neglect of rare events with major consequences like death.

Theorem 1 shows that Axiom 2 eliminates those examples that Arrow calls outrageous. We can also see how Axiom 2 provides a reasonable solution to the problem, as follows. Axiom 2 implies that there are catastrophic outcomes, such as the risk of death, so terrible that people are unwilling to accept a low probability of death to obtain one cent versus zero cents, no matter how low that probability may be. Indeed, according to the sensitivity Axiom 2, no probability of death is acceptable when one cent and 0 cents are involved. However according to Axiom 2, in some cases, the probability can be low enough to make the lottery involving death acceptable. As shown in Theorem 2, it all depends on what the other outcomes are. This becomes clear in our approach and seems a reasonable solution to the problem that Arrow raises.

For example, if in the above example “one cent were replaced by one billion dollars” – as Arrow (1966 p. 256, lines 31–32) suggests – under certain conditions we may be willing to choose the lottery that involves a low probability of death and one billion dollars over the lottery that offers 0 cents. Indeed, some of the subjects in the pill experiment state that they might have chosen to take the pill for a larger amount (“the amount is not big enough” (3 subjects), “the amount is too small to dramatically change my life”, “I would have answered ‘Yes’ if I was almost in absolute poverty”).

More to the point, consider the same type of death risk: a low probability of death caused by a medicine that can cure an otherwise incurable cancer may be preferable to no cure. A sick person may evaluate a cure – no matter how risky – higher than a healthy person would, and may be willing to take risks that a healthy person would not. In the same spirit, as shown in the pill experiment, the same individual may change his/her mind depending on factors exogenous to the outcomes. Here, among the two reasons that significantly explained such a change in the subjects that answered both 1998 and 2009 surveys, were “change in marital/familial status” and “change in the perception of the probability”.

The former has to do with the painful implications the death would have for relatives. The latter has to do with the subjective perception of the probability 10–9. One reason suggests itself: the 9/11 attacks, which occurred between the two experiments. Indeed, although subjects may have considered the outcome “simultaneous crashes of two commercial flights into the World Trade Center within the same hour” of tiny probability, the fact that it actually occurred may have led them to change their perception of tiny probability. No catastrophe had ever been so widely covered worldwide. This may explain that, in 2009 answers, new subjects do not significantly differ from previously 1998 surveyed subjects when surveyed in 2009. Indeed, Sunstein (2003) also provides evidence that individuals show unusually strong reactions to low-probability catastrophes especially when their emotions are intensely engaged. This “probability neglect” is also explored in Sunstein and Zeckhauser (2011) regarding both fearsome risks and the resulting damaging overreactions shown in individual behavior and government regulations.

Recently, Chanel and Chichilnisky (2009) report experimental results from a study of the predictions of the standard EU framework under catastrophic risks. Subjects faced choices among events involving “being locked up in a room with no chance of escaping, being freed or communicating (with relatives, friends...), with nothing interesting to do”. The events differed as to the duration of detention, and the catastrophic event was created by making the period of detention 40 years. Interestingly, the results obtained are in line with what obtained here: more than half the subjects did not behave according to EU theory whereas the remaining half answered according to EU theory; however all behaved according to the approach proposed in this article.

To conclude, the alternative approach proposed in this article further the treatment of catastrophic outcomes in two ways.

First, it provides a new measure for the value of life with two important characteristics: it values life more highly than under the EU criterion, and this value is shown to depend on factors, not only on the numerical value of what is being offered. This is because catastrophes are worst-case events whose weight depends on what else is going on in people’s lives.

Second, the alternative approach challenges the belief that EU properly expresses rationality in situations involving catastrophic outcomes. Indeed, EU theory is found to perform poorly in explaining the actual behavior of most of the subjects in the experiment, even though these subjects are fully familiar with the logic behind EU theory, while the alternative approach performs quite well. We do not reject the MC axiom outright — nor do we reject EU outright. Rather, we find that it is more realistic and satisfactory that MC be satisfied in some cases and not in others — EU axioms therefore being satisfied in some cases and not in others. Requiring that MC be satisfied in all cases and thus that EU utility axioms hold in all cases is problematic, since it implies insensitivity to rare and potentially catastrophic events. Hence, our approach stands as an alternative proposal for defining rational behavior in the face of catastrophes such as death. In any case, in the absence of rare events with major consequences, our theory of choice under uncertainty is consistent with and mirrors the standard EU theory (our Axiom 2 is indeed void of meaning), and can therefore be viewed as an extension of classic decision theory.

Finally, an interesting avenue for future research might be to explore how the brain works while considering outcomes involving

12 Note that EU could be used in certain cases to rationalize answers like the one we obtained, without providing a consistent set of axioms that create a well defined theory, by assuming that some subjects are infinitely averse to risk (unbounded below utility function). However, this is a somewhat ad hoc treatment not satisfactory theoretically since it brings back in the St Petersburg Paradox.

13 Or if the death event had been replaced by a less frightening event, say a €152,450 loss, we are willing to bet that most of the subjects in the pill experiment would have accepted the 10−9 probability of loss.
catastrophic and non-catastrophic events. Are the same zones activated? In the same order? For the same length of time? Functional magnetic resonance imaging or positron emission tomography should help answer these questions, since neuroeconomic decision science is no longer a utopian concept (see for instance Smith et al., 2002; or Knutson and Peterson, 2005).

Acknowledgments

This research is part of Columbia’s Program on Information and Resources, and the Columbia Consortium for Risk Management (CCRM). It was motivated by Olivier Chichilnisky’s experimental research and is based on earlier results in “The Topology of Fear” (Chichilnisky, 2000, 2002, 2009). It is partly supported by program Riskemotion (ANR-08-RISKNAT-007-01), which is gratefully acknowledged. We thank two anonymous referees, Marjorie Sweetko and Jean-Christophe Vergnaud for helpful comments and suggestions, congress participants at Montréal 2010 WCERE for helpful discussions, and the 127 former and current Greqam members for their kind participation.

Appendix. Countably and Purely Finitely Additive Measures

The space of continuous linear functions on $L_\infty$ is a well known space called the “dual” of $L_\infty$ and is denoted $L_\infty^\ast$. This dual space has been fully characterized e.g. in Yosida and Hewitt (1952) or Yosida (1954). Its elements are defined by integration with respect to measures on $R$. The dual space $L_\infty^\ast$ consists of $(1) \, f$ functions $g$ that define countably additive measures $\mu$ on $R$ by the rule $\mu(A) = \int_A f(x) \, dx \text{ where } \int f(x) \, dx < \infty \text{ and therefore } f \text{ is absolutely continuous with respect to the Lebesgue measure, namely it gives measure zero to any set with Lebesgue measure zero, }$ and $(ii) \, a ‘non-L_1’ \text{ part consisting of purely finitely additive measures } \rho \text{ that are ‘singular’ with respect to the Lebesgue measure and give positive measure to sets of Lebesgue measure zero; these measures } \rho \text{ are finitely additive but they are not countably additive. A measure } \eta \text{ is called \textit{finitely additive} when for any family of pairwise disjoint measurable sets } \{A_i\}_{i=1}^N \Rightarrow \sum \eta(A_i) \Rightarrow N \text{ and } \eta(\bigcup_{i=1}^N A_i) = \sum_{i=1}^N \eta(A_i). \text{ The measure } \eta \text{ is called \textit{countably additive} when for any family of pairwise disjoint measurable sets } \{A_i\}_{i=1}^\infty \Rightarrow \sum \eta(A_i) \Rightarrow \sum_{i=1}^\infty \eta(A_i). \text{ The countably additive measures are in a one-to-one correspondence with the elements of the space } L_1(R) \text{ of integrable functions on } R. \text{ However, purely finitely additive measures cannot be identified by such functions. Yet purely finitely additive measures play an important role, since they ensure that the ranking criteria are ‘sensitive to rare events’ (Axiom 2). These measures define continuous linear real valued functions on } L_\infty, \text{ thus belonging to the dual space of } L_\infty \text{ (Yosida, 1974), but cannot be represented by functions in } L_1.$

References


